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Konishi anomaly approach to gravitational F-terms

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Abstract: We study gravitational corrections to the effective superpotential in theories with a single adjoint chiral multiplet, using the generalized Konishi anomaly and the gravitationally deformed chiral ring. We show that the genus one correction to the loop equation in the corresponding matrix model agrees with the gravitational corrected anomaly equations in the gauge theory. An important ingrediant in the proof is the lack of factorization of chiral gauge invariant operators in presence of a supergravity background. We also find a genus zero gravitational correction to the superpotential, which can be removed by a field redefinition.

Keywords: M(atrix) Theories, Anomalies in Field and String Theories, Supersymmetric Effective Theories.

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1. Introduction

Recently much progress has been made in the computation and understanding of F-terms which describe the coupling of $N = 1$ gauge theories to $N = 1$ supergravity [1]–[5]. In particular the original conjecture of Djikgraaf and Vafa [1], relating these F-terms to non-planar corrections to the free energy of a related matrix model has been proved by [5] using diagrammatic techniques, extending to the gravitational case the super-Feynman diagram techniques of [6] Crucial ingredient in the proof was the modification of the chiral ring relations due to the coupling of the gauge theory to supergravity. In particular, if one restricts to the first non-trivial gravitational F-term contribution, corresponding to the genus one correction in the related matrix model, one needs to take into account just the modification which follows from standard $N = 1$ supergravity tensor calculus. The purpose of the present work is rather to study the problem from the viewpoint of generalized Konishi anomaly relations in the chiral ring, by extending to the case of $N = 1$ gauge theories coupled to $N = 1$ supergravity the strategy of [7] The key point in our analysis will be,together with the modification of the chiral ring mentioned above, the observation that in the presence of a non-trivial supergravity background the usual factorization property of chiral correlators does not hold. In particular connected two point functions are generically non-vanishing, much like in matrix models, where connected correlators receive a subleading $1/N^2$ contribution, in the $1/N$ expansion

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The paper is organized as follows: in section 2 we study in detail the deformation of the chiral ring due to the coupling to supergravity and some of the resulting relations that we will need subsequently. In section 3 we estimate the chiral, connected two- and three-point functions that will enter later in the equations obtained using the generalized Konishi anomaly. Section 4 contains the main results: we derive there the generalized loop equations which enables us to solve for the relevant connected two-point functions. We show the uniqueness of the solution and use it to compute the first non-trivial gravitational correction to the effective superpotential.. We finally show that this agrees with the genus one correction to the matrix model free energy. In section 5 we make our conclusions and mention some open problems.

2. The chiral ring

A basic ingredient in deriving the effective F-terms in $N = 1$ gauge theories after integrating out the adjoint matter is the chiral ring [7]. All quantities in the chiral ring are defined modulo $\bar{D}_{\dot{\alpha}}$ exact terms, here $\bar{D}_{\dot{\alpha}}$ refers to the differential operator conjugate to the supersymmetric current $\bar{Q}_{\dot{\alpha}}$. It is sufficient to restrict the quantities to be in the chiral ring as $\bar{D}_{\dot{\alpha}}$ exact terms do not contribute to F terms. For the $N = 1$ gauge theory with adjoint matter on R^4 the chiral ring relations are given by

$$[W_\alpha, \Phi] = 0 \bmod \bar{D}\,, \qquad \{W_\alpha, W_\beta\} = 0 \bmod \bar{D}\,, \tag{2.1}$$

where W_α is the $N = 1$ gauge multiplet and Φ is the chiral multiplet in the adjoint representation of the gauge group.

The chiral ring relations given above are modified in the presence of a curved background [5]. We review here the derivation of the modified chiral ring in presence of background $N = 1$ gravity. Consider the following $\bar{D}$ exact quantity

$$\{\bar{D}^{\dot{\alpha}}, [D_{\alpha\dot{\alpha}}, W_\beta]\} = \{[\bar{D}^{\dot{\alpha}}, D_{\alpha\dot{\alpha}}], W_\beta\}\,, \tag{2.2}$$

where $D_{\alpha\dot{\alpha}}$ is the full covariant derivative containing the gauge field and the spin connection. We used the Jacobi identity and the fact that W_β is chiral to obtaining the second term in the above equality. From the Bianchi identity [8] for covariant derivatives one has

$$[\bar{D}^{\dot{\alpha}}, D_{\alpha\dot{\alpha}}] = 4iW_\alpha - 8iG_{\alpha\beta\gamma}M^{\beta\gamma}\,, \tag{2.3}$$

where $G_{\alpha\beta\gamma}$ is the $N = 1$ Weyl multiplet and $M^{\alpha\beta}$ refers to the Lorentz generator on spinors, its action on a spinor is given by

$$[M^{\alpha\beta}, \psi_\gamma] = \frac{1}{2}(\delta^\alpha_\gamma\psi^\beta + \delta^\beta_\gamma\psi^\alpha)\,. \tag{2.4}$$

Substituting the Bianchi identity (2.3) in (2.2) and using the action of the Lorentz generator we find the deformed chiral ring given below

$$\{W_\alpha, W_\beta\} = 2G_{\alpha\beta\gamma}W^\gamma \bmod \bar{D}\,, \qquad \{W^\alpha, \Phi\} = 0 \bmod \bar{D}\,. \tag{2.5}$$

The second equation is obtained by replacing the W_β in (2.2) by Φ. For the conventional $N = 1$ supergravity theory, in the first equation above, only the $\mathrm{SU}(N)$ part of the gauge field W^γ appears in the right hand side. In [5], the modification of the ring involved the $\mathrm{U}(1)$ part of the gauge field as well, which corresponds to a non-standard $N = 1$ supergravity theory relevant to D-brane gauge theories. In the present paper we will always be restricting ourselves to the standard $N = 1$ supergravity. In order to avoid explicitly including the $\mathrm{SU}(N)$ projectors in all the formulae below, we shall always take gauge field backgrounds to be in the $\mathrm{SU}(N)$ part of $\mathrm{U}(N)$.

Using the deformed chiral ring we can derive many identities valid in the ring which are used crucially in the next sections[1]. From the definition of W^2 and (2.5) we have

$$\begin{aligned} W_\alpha W_\beta &= \frac{\epsilon_{\alpha\beta}}{2} W^2 + G_{\alpha\beta\gamma} W^\gamma \,, \\ W_\alpha W_\beta W_\gamma &= \frac{1}{2}\epsilon_{\alpha\beta} W^2 W_\gamma + \frac{1}{2} G_{\alpha\beta\gamma} W^2 + G_{\alpha\beta\delta} G^\delta{}_{\gamma\sigma} W^\sigma \,, \end{aligned} \tag{2.6}$$

using the above identities we are led to the following relations

$$\begin{aligned} W_\alpha W^2 &= -\frac{1}{2} W^2 W_\beta - \frac{1}{2} G^2 W_\alpha \,, \\ W^2 W_\alpha &= -\frac{1}{2} W_\alpha W^2 - \frac{1}{2} G^2 W_\alpha \,, \\ [W^2, W_\alpha] &= 0 \,, \qquad W^2 W_\alpha = -\frac{1}{3} G^2 W_\alpha \,, \qquad W^2 W^2 = -\frac{1}{3} G^2 W^2 \,, \\ (G^2)^2 &= G^{\alpha\beta\gamma} G_{\alpha\delta\sigma} G^\sigma{}_{\beta\rho} G^{\delta\rho}{}_\gamma \,. \end{aligned}$$

These identities imply that the gauge invariant combination of certain chiral operators vanish in the chiral ring. The following chiral operator vanishes in the chiral ring.

$$G_{\alpha\beta\gamma} \operatorname{Tr}(W^\gamma \Phi\Phi \ldots) = 0 \,, \bmod \bar{D} \,. \tag{2.7}$$

It is clear that if there are no Φ's in the trace, the above equation is true for the gauge group $\mathrm{SU}(N)$. To proof the above identity for arbitrary number of Φ we use the following equation

$$\begin{aligned} \operatorname{Tr}(W_\alpha W_\beta \Phi\Phi \cdots) &= -\operatorname{Tr}(W_\beta W_\alpha \Phi\Phi \cdots) \,, \\ &= \frac{1}{2}\epsilon_{\alpha\beta} \operatorname{Tr}(W^2 \Phi\Phi \cdots) \,. \end{aligned} \tag{2.8}$$

To obtain the first equation above we have used the cyclic property of trace and (2.5). Now multiplying the first equation of (2.6) with arbitrary number of Φ's and using (2.8) we obtain (2.7). Multiplying (2.7) by $G^{\alpha\beta\delta}$ and using (A.2) we obtain

$$G^2 \operatorname{Tr}(W_\alpha \Phi\Phi \ldots) = 0 \,, \bmod \bar{D} \,. \tag{2.9}$$

Another important identity in the chiral ring is

$$G^4 = (G^2)^2 = 0 \,, \bmod \bar{D} \,. \tag{2.10}$$

[1] The conventions followed in this paper are given in appendix A.

The proof goes along the same lines as the derivation of the deformed chiral ring. Consider the following $\bar{D}$ exact quantity

$$\{\bar{D}^{\dot{\alpha}}, [D_{\alpha\dot{\alpha}}, G_{\beta\gamma\delta}]\} = \{[\bar{D}^{\dot{\alpha}}, D_{\alpha\dot{\alpha}}], G_{\beta\gamma\delta}\}\,. \tag{2.11}$$

As $G_{\beta\gamma\delta}$ is uncharged with respect to gauge field the covariant derivative $D_{\alpha\dot{\alpha}}$ contains only the spin connection. The Bianchi identity for covariant derivatives now implies

$$[\bar{D}^{\dot{\alpha}}, D_{\alpha\dot{\alpha}}] = -8iG_{\alpha\beta\gamma}M^{\beta\gamma}\,. \tag{2.12}$$

Substituting the above equation in (2.11) we obtain the following equation in the chiral ring

$$G_{\alpha\beta\sigma}G^{\sigma}{}_{\gamma\delta} + G_{\alpha\gamma\sigma}G^{\sigma}{}_{\beta\delta} + G_{\alpha\delta\sigma}G^{\sigma}{}_{\beta\gamma} = 0\,. \tag{2.13}$$

Multiplying this equation by $G^{\alpha\gamma\rho}G^{\beta\delta}{}_{\rho}$ so that all the free indices are contracted and using the last equation in (2.7) and (A.2) we obtain (2.10). As a result the gravitational corrections to the F-terms truncate at order G^2.

3. Estimates of connected part of correlators

In the absence of gravity correlators of gauge invariant operators factorize in the chiral ring [7]. This factorization enabled one to show that the loop equations satisfied by the resolvent on the gauge theory agreed with the loop equations of the matrix model in the large N limit. On the matrix model side factorization in the loop equations was due to the the large N limit. However the correspondence of the gauge theory with the matrix model proposed by Dijkgraaf and Vafa in [1] goes beyond the large N approximation. If the $N = 1$ gauge theory is placed in a background gravitational field then the gravitational corrections to the F terms of the gauge theory is of the form

$$\Gamma_1 = \int d^4x d^2\theta G^2 F_1(S)\,, \tag{3.1}$$

here S refers to the gaugino condensate. The Dijkgraaf and Vafa proposal states that the one can calculate F_1 from the genus one diagrams of the corresponding matrix model. It has been shown in [5] that the modification of the chiral ring in the presence of gravity allows the computation of the gravitational corrections to the F-term. The gravitational corrections enter at genus one on the gauge theory side and they reduce to the genus one diagrams of the corresponding matrix model. This implies that the loop equations of the gauge theory would not factorize in presence of gravity, as the loop equations of the matrix model do not factorize at genus one. Therefore a priori we expect that gauge invariant operators do not factorize in the presence of gravity. In this section we estimate the gravitational corrections to the connected parts of various correlators that can appear in the loop, equations using the deformation of the chiral ring in presence of gravity (2.5).

To estimate the gravitational corrections to the F-term obtained by integrating out the chiral multiplet we require the two point function the chiral scalars $\langle\Phi\Phi\rangle$. This is obtained after integrating out the antichiral scalar and it is given by

$$\langle\Phi(x,\theta)\Phi(x',\theta')\rangle = \frac{\bar{m}}{-\Box + \frac{i}{2}D^{\alpha}W_{\alpha} + iW^{\alpha}D_{\alpha} + m\bar{m}}\delta^4(x-x')\delta^2(\theta-\theta')\,. \tag{3.2}$$

This propagator in the presence of a gravitational background was derived in [9] and in writing the above equation we have assumed that the gravitational background is on shell which allows one to set the other terms which occur in the propagator to zero, $\Box$ is (3.2) stands of the full gauge and gravitational covariant derivative. The action of W_α is dictated by the representation of Φ, in this paper we will restrict ourselves to the adjoint action. The delta functions in (3.2) refer to the full covariant delta function in curved superspace. In order to obtain the leading estimates for the connected component of various correlators due to the modification of the chiral ring it is sufficient to use the free d' alembertian operator and a constant gaugino background. We argue this as follows, we can expand the propagator of (3.2) in a weak field as

$$\begin{aligned}\frac{\bar{m}}{-\Box + iD^\alpha W_\alpha + m\bar{m}} &= \frac{\bar{m}}{-\Box_0 + iW^\alpha D_\alpha + m\bar{m}} + \\ &+ \frac{\bar{m}}{-\Box_0 + iW^\alpha D_\alpha + m\bar{m}}(\Box - \Box_0)\frac{1}{-\Box_0 + iW^\alpha D_\alpha + m\bar{m}} + \cdots\end{aligned} \tag{3.3}$$

here $\Box_0$ refers to the free d'alembertian operator. We have also dropped the terms $D^\alpha W_\alpha$ in (3.2) as we have considered a covaraintly constant gaugino background. From the expansion we see that the corrections in using the free d' alembertian operator in the propagator always occur with the factor $(\Box - \Box_0)$, which is proportional to the gravitational background and therefore subleading to the estimate obtained using the free d' alembertian operator in the first term. One has to make a similar expansion for the covariant delta function in (3.2) and again one can see corrections in using the flat space delta function are subleading. However we will see later that if there is no gravitational contribution form the deformed chiral ring the leading estimate for the connect part of correlators arise from corrections in the propagator due to the presence of the full covariant $\Box$ and the covariant delta function. For the free d'alembertian operator in the propagator it is possible to go over to momentum space and to write a Schwinger parameterization of the propagator as follows.

$$\langle \Phi(x,\theta)\Phi(x',\theta)\rangle = \int ds d^4p d^2\pi e^{ip(x-x')} e^{-\frac{s}{\bar{m}}(p^2 + W^\alpha \pi_\alpha + m\bar{m})}, \tag{3.4}$$

where $\pi^\alpha = iD^\alpha$. In the above equation we have restricted to the superspace variable θ to be the same at x and x' as we will be interested in correlators at the same point in the superspace variable θ. We will now use this propagator and the modified chiral ring (2.5) to make estimates for the connected part of various correlators. The modified ring allows more than two insertions of W_α in a given index loop, using the identities in (2.7) such contributions can be converted to gravitational corrections. At this point one might wonder if contributions to the connected diagrams of gauge invariant operators in presence of a gravitational background are in contradiction with the result found in [10]. There it was found that on an arbitrary Kähler manifold gauge invariant operators of the $N = 1$ theory factorize. The background considered in [10] was entirely bosonic, we find the estimates of contribution to the connected diagram indeed vanish for a purely bosonic background, thus there is no contradiction with [10]. We will indicate this as we evaluate the estimates of various correlators.

The various operators involved in the correlators of interest are

$$\begin{aligned}
\mathcal{R}(z)_{ij} &= -\frac{1}{32\pi^2}\left(\frac{W^2}{z-\Phi}\right)_{ij}, & R(z) &= \mathrm{Tr}\,\mathcal{R}(z) \\
\rho_{\alpha\ ij}(z) &= \frac{1}{4\pi}\left(\frac{W_\alpha}{z-\Phi}\right)_{ij}, & w_\alpha(z) &= \mathrm{Tr}\,\rho_\alpha(z) \\
\mathcal{T}(z)_{ij} &= \left(\frac{1}{z-\Phi}\right)_{ij}, & T(z) &= \mathrm{Tr}\,\mathcal{T}(z)
\end{aligned} \tag{3.5}$$

here we have defined separate symbols for the matrix elements and the trace for later convenience, the gauge invariant operators we will consider are the ones with the trace in the above equation. The contour integrals of R, w_α and T around i-th branch cut define the gaugino bilinear S_i, the U(1) gauge field $w_{\alpha i}$ and N_i respectively in U(N_i) subgroup of U(N) as in [7]. The fact that we are here restricting the background gauge field to be in SU(N) rather than U(N) implies that $\sum_i w_{\alpha i} = 0$. The chiral ring relation $G^2 w_\alpha(z) = 0$ implies that $G^2 w_{\alpha i} = 0$ for all i.

We first consider estimates of the connected part of two point function, we will discuss in detail the estimate for the follow correlator

$$\langle R(z,x,\theta), R(w,y,\theta)\rangle_c\,, \tag{3.6}$$

where the subscript stands for the connected part and out line the derivation of the estimates for the other two point functions. The various contribution to this correlator in (3.6) can be found by expanding in z and w, by definition of the connected correlator the expansion starts of with the power $1/z^2w^2$. Let us focus on a Feynman diagram consisting of l loops, there will be $l+1$ bosonic and fermionic momentum integrations in this diagram. The extra momentum integral comes from the final Fourier transform to convert to the position space representation for the above correlator. We can organize this diagram into index loops due to the adjoint action of W_α. The fermionic momentum integral forces us to bring down $2(l+1)$ powers of W from the propagator in (3.4). To obtain the leading gravitational correction we would like the number of index loops to be as large as possible so that we can avoid having more than two W's in a given index loop. The number of index loops h and the number of loops are related by $l = h - 1 + 2g$, where g is the genus of the diagram. For a given number of Feynman loops the number of index loops is largest for genus zero, thus the leading estimate to the connected graph arises from the planar diagram. For a planar diagram we need to saturate the fermionic momentum integrals by bringing down $2h$ W's. This can be done by inserting W^2 in h index loops. We still have two more external W^2 in (3.6). This can at best be inserted in two different index loops. Thus we have two index loops with $(W^2)^2$ insertions. Using the identities in (2.7) we see that each of them reduces to G^2W^2. Thus there is a term proportional to G^4W^2 on one of the index loops. Note that if one had a purely bosonic background both G and W would start at θ in the superspace expansion, thus G^4W^2 would vanish in agreement with [10]. In fact G^4W^2, as is trivial in the chiral ring by (2.10), the leading estimate for the correlator in (3.6) vanishes. In the next section it is shown that the two point function in (3.6) in fact vanishes. Next we consider the following two point function

$$\langle R(z,x,\theta) w_\alpha(w,x',\theta)\rangle_c\,, \tag{3.7}$$

we can apply the same counting again, finally in the planar diagram we will be left with at best with one index loop with $(W^2)^2$ and one with W^2W_α insertions. This reduces to a G^2W^2 insertion and a G^2W_α insertions, which implies that the leading gravitational contribution to (3.7) is proportional to G^4. Now we have seen in (2.9) that G^2W_α is zero in the chiral ring, thus this leading estimate in fact vanishes in the chiral ring. Similarly, consider the correlator

$$\langle R(z,x,\theta) T(w,x',\theta)\rangle_c\,. \tag{3.8}$$

Here we will be left with $(W^2)^2$ in a single index loop, which reduces to G^2W^2. Thus the above correlator is proportional to G^2W^2. For a purely bosonic background we see that this contribution again vanishes, consistently with [10]. For the case of

$$\langle w^\alpha(z,x,\theta) w_\alpha(w,x',\theta)\rangle_c\,, \tag{3.9}$$

we will be left with either $W^\alpha W^2$ insertion in two different index loops or a $(W^2)^2$ insertion in a single index loop. The former case vanishes in the chiral ring, but the latter case survives, with a contribution proportional to G^2W^2. For the following two point function

$$\langle w_\alpha(z,x,\theta) T(w,x',\theta)\rangle_c\,, \tag{3.10}$$

there is a $W^\alpha W^2$ insertion in a single index loop, which is proportional to G^2W_α. Note that this leading contribution vanishes in the chiral ring due to (2.9) and also for a purely bosonic background. Finally, we have the two point function

$$\langle T(z,x,\theta) T(w,x',\theta)\rangle_c\,. \tag{3.11}$$

For this case all the h index loops are saturated with one W^2 and there are extra insertions of W^2 for any of the h index loops, as there is no external W. Thus we have no contribution for the connected part of this correlator from the modified ring. However we will see later that there is a direct gravitational contribution to the above correlator. This can be seen roughly as follows: the d' alembertian in (3.2) carries the covariant derivatives which can possibly contribute to the connected two point function, as seen in the expansion of the propagator in (3.3). This fact can be further justified by the evaluation of the 1-loop effective action obtained by integrating out the chiral multiplet in the absence of the gauge field background, which gives a term proportional to $G^2\ln(m)$ [9]. Therefore we expect the leading term in the correlator in (3.11) to be proportional to G^2 and this will be shown explicitly in the next section. Again we see that for a purely bosonic background G^2 is proportional to θ^2, which implies that the lowest components of the superfields in (3.11) factorize consistently with [10].[2] We summarize the estimates of the various connected two point correlators in the following table for future reference where S represents schematically any of the S_i's, and we have used the chiral ring relations $G^4=0$ and $G^2w_{\alpha i}=0$.

[2]Note that the fact that the lowest component factorize cannot be used to promote it to a superfield equation as the is no Q_α which preserves the background.

Now we provide estimates for the fully connected part of various three point functions. All the fully connected part of the three point functions are proportional to at least G^4, therefore using (2.10) they all vanish in the chiral ring. We discuss the method of arriving at the estimates for one case in detail and just outline the results for the others. Consider the following fully connected three point function.

$$\langle R(z,x,\theta)R(w,x',\theta)R(u,x'',\theta)\rangle_c\,. \tag{3.12}$$

Correlator	Estimate
$\langle RR\rangle_c$	$G^4=0$
$\langle Rw_\alpha\rangle_c$	$G^4=0$
$\langle RT\rangle_c$	G^2S^h
$\langle w^\alpha w_\alpha\rangle_c$	G^2S^h
$\langle w_\alpha T\rangle\rangle_c$	$G^2S^{h-1}w_{\alpha\ i}=0$
$\langle TT\rangle_c$	G^2S^{h-1}

Table 1: Estimates for connected correlators.

By the definition of the full connected three point function, the first possibly non zero term of the expansion in z,w,u starts at $1/(zwu)^2$. Consider a contribution to any of the correlators appearing in this expansion. A Feynman diagram consisting of l loops will now have $l+2$ bosonic and fermionic momentum integrations. This is because a three point function in momentum space will in general have two external independent momenta, and then converting that to position space will involve these additional momentum integrals. As we have argued earlier for the case of the two point function, the leading contribution will be from the genus zero graphs. For a planar graph then there are $2(h+1)$ fermionic momentum integrals to be done. Therefore in addition to inserting h index loops by W^2, at best four different index loops will have $(W^2)^2$ insertions (we assume l is large enough). Using the identities in (2.7) we see that each $(W^2)^2$ insertion is proportional to G^2W^2. Thus the above three point function is proportional to G^8, but G is fermionic and has 4 independent components, thus G^8 vanishes due to Fermi statistics. Similar arguments show that the following correlators vanish

$$\begin{aligned}
\langle R(z,x,\theta)R(w,x',\theta)w_\alpha(u,x'',\theta)\rangle_c &= 0\,,\\
\langle R(z,x,\theta)R(w,x',\theta)T(u,x'',\theta)\rangle_c &= 0\,,\\
\langle R(z,x,\theta)w^\alpha(w,x',\theta)w_\alpha(u,x'',\theta)\rangle_c &= 0\,,\\
\langle R(z,x,\theta)w^\alpha(w,x',\theta)T(u,x'',\theta)\rangle_c &= 0\,.
\end{aligned}$$

The first correlator in (3.13) is proportional to G^8 and the rest are proportional to G^6, thus they vanish due to Fermi statistics. Now consider the following three point function

$$\langle w^\alpha(z,x,\theta)T(w,x',\theta)T(u,x'',\theta)\rangle_c\,. \tag{3.13}$$

We have seen that in a planar graph the fermionic momentum integrations force one to insert at least one factor of W^2 in all of the h index loops and there is at least one index loop with a $(W^2)^2$ insertion. Using the identities in (2.7) this can be manipulated to a gravitational contribution proportional to G^2W^2. For the above correlator there is one more index loop with an insertion of W^2W^α and again using the identities in (2.7), this term is proportional to G^2W^α. Thus the leading gravitational contribution to the three point function in (3.13) is proportional to G^4W_α and thus it is zero in the chiral ring using (2.9). Next we consider the following three point function

$$\langle R(z,x,\theta)T(w,x',\theta)T(u,x'',\theta)\rangle_c\,. \tag{3.14}$$

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As discussed above, since there are $2(h+1)$ momentum integration, all the h index loops have at least a W^2 insertion with one having a $(W^2)^2$ insertions, the above correlator has an external W^2. Therefore the leading gravitational contribution arises with two different index loops each with a $(W^2)^2$ insertion and using the identities in (2.7) this can be manipulated in the chiral ring to give a factor of G^4. The following three point function is also proportional to G^4

$$\langle W^\alpha(z,x,\theta) W_\alpha(w,x',\theta) T(u,x'',\theta)\rangle_c \,. \tag{3.15}$$

Here again due to the momentum integrations there is already a factor of G^2, the two external W's can be inserted in another index loop, but this loop already has an insertion of W^2, which gives rise to a contribution proportional to G^2. Thus the three point function in (3.15) is proportional to G^4. Therefore the correlators in (3.14) and (3.15) vanish in the chiral ring due to (2.10). Finally, let us consider the following three point function

$$\langle T(z,x,\theta) T(w,x',\theta) T(u,x'',\theta)\rangle_c \,. \tag{3.16}$$

As in the case for (3.11) we can not estimate the G dependence of this correlator solely using the chiral ring. But from the fact that the correlator in (3.11) is proportional to G^2 and since there is at least one index loop with $(W^2)^2$ insertion, we can arrive at the conclusion that the above three point function will be proportional to G^4. To sum up we have examined all possibly non-vanishing full connected three point fully and found them to be least proportional to G^4, and therefore using (2.10) they vanish in the chiral ring.

4. Anomaly equations and matrix model loop equations

In this section we will use the generalized Konishi anomaly to extract the gravitational corrections to the effective superpotential. We recall that the anomaly equations obtained in [7] in the absence of gravitational fields are as follows

$$\begin{aligned}
\langle R(z)R(z)\rangle - \langle \mathrm{tr}(V'(\Phi)\mathcal{R}(z))\rangle &= 0\\
2\langle R(z)w_\alpha(z)\rangle - \langle \mathrm{tr}(V'(\Phi)\rho_\alpha(z)\rangle &= 0\\
2\langle R(z)T(z)\rangle - \langle \mathrm{tr}(V'(\Phi)\mathcal{T}(z)\rangle + \langle w^\alpha(z)w_\alpha(z)\rangle &= 0
\end{aligned} \tag{4.1}$$

Here V denotes the classical superpotential of degree $n+1$. We have indicated above the full two point functions that include the disconnected and connected two point functions. The latter vanish in the absence of gravitational field but as we will show below do not vanish in the presence of the gravitational field. These equations were obtained by the generalized Konishi anomalies upon transforming the adjoint chiral field Φ as $\delta\Phi_{ij}$ equal to $\mathcal{R}_{ij}(z)$, $\eta^\alpha\rho_{\alpha\ ij}(z)$ and $\mathcal{T}_{ij}(z)$ respectively, with η^α being an arbitrary field independent spinor. In general for the infinitesimal transformation $\delta\Phi_{ij}=f_{ij}$, the generalized Konishi anomaly is given by

$$\frac{\delta f_{ji}}{\delta\Phi_{k\ell}} A_{ij,k\ell} \,. \tag{4.2}$$

In the absence of gravitation

$$A_{ij,k\ell} = (W^2)_{kj}\delta_{i\ell} + \delta_{kj}(W^2)_{i\ell} - 2W^{\alpha}_{kj}W_{\alpha i\ell}\,. \tag{4.3}$$

Using the above anomaly and the equation $\{W_\alpha, W_\beta\} = 0$ in the chiral ring, one obtains the equation (4.1).

In the presence of the gravitational field $G_{\alpha\beta\gamma}$, these equations are modified for two reasons: firstly there is a direct gravitational anomaly (ie. even in the absence of gauge fields, in other words when chiral multiplets couple only to gravitational fields) and secondly due to the modification of the ring (2.5).

Under the infinitesimal transformation $\delta\Phi_{ij} = f_{ij}$ the pure gravitational contribution to the anomaly is

$$\frac{\delta f_{ji}}{\delta\Phi_{k\ell}}\alpha G^2\delta_{kj}\delta_{i\ell}\,, \tag{4.4}$$

where $\alpha = 1/3$ as shown in the appendix B, and from now on G^2 will denote $\frac{1}{32\pi^2}G^{\alpha\beta\gamma}G_{\alpha\beta\gamma}$. This changes

$$A_{ij,k\ell} \to A_{ij,k\ell} + \frac{1}{3}G^2\delta_{kj}\delta_{i\ell}\,. \tag{4.5}$$

It is easy to see that the pure gravitational anomaly and the modification of the chiral ring (2.5) together with the consequent identities given in the equations (2.7), (2.9), (2.10), give rise to the following modification of the equations (4.1):

$$\begin{aligned} \langle R(z)R(z)\rangle - \langle \mathrm{Tr}(V'(\Phi)\mathcal{R}(z))\rangle &= 0 \\ 2\langle R(z)w_\alpha(z)\rangle - \langle \mathrm{Tr}(V'(\Phi)W_\alpha(z))\rangle &= 0 \\ 2\langle R(z)T(z)\rangle - \langle \mathrm{Tr}(V'(\Phi)\mathcal{T}(z))\rangle + \langle w^\alpha(z)w_\alpha(z)\rangle - \frac{1}{3}G^2\langle T(z)T(z)\rangle &= 0\,. \end{aligned} \tag{4.6}$$

Note that in the first two equations above the pure gravitational anomaly cancels with the contributions coming from the modification of the chiral ring via eq. (2.7). This happens due to the remarkable fact that the pure gravitational anomaly (4.5) comes with a factor of $\frac{1}{3}$!! We have also used the fact that $G^2 w_\alpha$ vanishes in the chiral ring. In the last equation however there is no contribution due to the modification of the ring and hence the last two terms on the left hand side arise solely from the pure gravitational anomaly. Here again, a priori, the two point functions are the sum of connected and disconnected parts.

For later purposes let us write the first equation in (4.6) more explicitly

$$\langle R(z)\rangle^2 - V'(z)\langle R(z)\rangle - \frac{1}{4}f(z) = -\langle R(z)R(z)\rangle_c\,, \tag{4.7}$$

where $f(z)$ is a polynomial of degree $n-1$ defined by

$$\langle \mathrm{Tr}(V'(\Phi) - V'(z))\mathcal{R}(z)\rangle = \frac{1}{4}f(z) \tag{4.8}$$

and the subscript c denotes the connected part of the correlation function. The latter, as we have seen in the last section, goes as G^4 and therefore trivial in the chiral ring. As a result the equation for R is unmodified by the gravitational field. Since the finite polynomial f is determined completely by the periods of R, i.e. the contour integrals around the various

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branch cuts C_i, $i = 1, \ldots, n$,

$$\frac{1}{2\pi i}\int_{C_i} dz R(z) = S_i \tag{4.9}$$

we conclude that R does not receive any gravitational corrections.

The strategy now is to expand all of the quantities appearing above in a perturbation series in G^2. Of course, this series ends at order G^2 since G^4 is trivial. Thus for example we write

$$\langle T\rangle = T^{(0)} + G^2 T^{(1)} \tag{4.10}$$

and similarly for the connected parts of the 2-point functions appearing in the above equations. As discussed in the last section, the latter start from order G^2, and therefore the equations for $T^{(0)}$ and $w_\alpha^{(0)}$ are the same as in [7]. To go beyond this we need to solve for the connected two point functions.

4.1 Equations for the connected two point functions in the presence of gravitational fields

We will now derive equations for the connected two point functions that appear in eq. (4.6). Although, in eq. (4.6) the connected 2-point functions are of the form $\langle R(z)T(z)\rangle$, i.e. both the operators are at same z, it turns out to be more convenient to consider the two operators at different points in the complex plane (say z and w). The reason is that we can impose conditions on a connected 2-point function, like $\langle R(z)T(w)\rangle$ that their integrals around various branch cuts in z and w plane vanish separately. As a result, we will be able to solve completely the corresponding generalized Konishi anomaly equations.

We illustrate the general method of obtaining the generalized Konishi anomaly equations for the connected 2-point functions in one example and then give the complete set of equations which can easily be derived following the methods given below.

Consider the infinitesimal transformation (local in superspace coordinates $(x^\mu, \theta, \bar\theta)$)

$$\delta\Phi_{ij} = \mathcal{R}_{ij}(z)T(w)\,. \tag{4.11}$$

The jacobian of this transformation has two parts

$$\frac{\delta(\delta\Phi_{ji})}{\delta\Phi_{kl}} = \frac{\delta R_{ji}(z)}{\delta\Phi_{kl}}T(w) + \sum_m R_{ji}(z)T_{mk}(w)T_{lm}(w) \tag{4.12}$$

The first term in the equation above together with the variation of the classical superpotential gives rise to

$$\begin{aligned}\langle (R(z)R(z) - \mathrm{Tr}(V'(\Phi)\mathcal{R}(z)))T(w)\rangle \;&= \langle (R(z)R(z) - \mathrm{Tr}(V'(\Phi)\mathcal{R}(z)))\rangle\langle T(w)\rangle\, + \\ &\quad + 2\langle R(z)\rangle\langle R(z)T(w)\rangle_c - \langle \mathrm{Tr}(V'(\Phi)\mathcal{R}(z)))T(w)\rangle_c + \\ &\quad + \langle R(z)R(z)T(w)\rangle_c \end{aligned} \tag{4.13}$$

where the subscript c denotes completely connected 2 or 3 point functions as indicated. The first term on the right hand side vanishes by virtue of the first equation of (4.6).

The second term in the jacobian when combined with the anomaly (4.3), (4.5) gives rise to a single trace contribution

$$-\frac{1}{3}G^2\langle\mathrm{Tr}(\mathcal{R}(z)\mathcal{T}(w)\mathcal{T}(w))\rangle = -\frac{1}{3}G^2\partial_w\frac{\langle R(z)\rangle - \langle R(w)\rangle}{z-w}\,. \tag{4.14}$$

Combining eqs.(4.13), (4.14) and the first equation of (4.6), one obtains the following equation for the connected correlation functions:

$$(2\langle R(z)\rangle - I(z))\langle R(z)T(w)\rangle_c + \langle R(z)R(z)T(w)\rangle_c - \frac{1}{3}G^2\partial_w\frac{\langle R(z)\rangle - \langle R(w)\rangle}{z-w} = 0\,. \tag{4.15}$$

Here the integral operator $I(z)$ denotes the following:

$$I(z)A(z) = \frac{1}{2\pi i}\int_{C_z} dy\frac{V'(y)A(y)}{y-z} \tag{4.16}$$

with the contour C_z encircling z and ∞. It is clear that for A equal to $\mathcal{R}$, ρ_α or $\mathcal{T}$, the integral operator reduces to

$$I(z)A(z) = V'(\Phi)A(z) \tag{4.17}$$

as is familiar in the matrix model works.

Since the last term in the eq. (4.15) is of order G^2 and involves one point function of R which is certainly not zero, we conclude that the sum of the remaining terms which involve connected correlations functions cannot all vanish at order G^2. This proves our basic assertion. In fact the connected 3-pt. function $\langle R(z)R(z)T(w)\rangle_c$ vanishes, as argued in eq. (3.13) in the last section, so eq. (4.15) implies that the connected 2-pt function $\langle R(z)T(w)\rangle_c$ does not vanish at order G^2.

In order to completely solve the relevant connected correlation functions, we need to consider all transformations of the form

$$\delta\Phi_{ij} = A_{ij}(z)B(w)\,, \tag{4.18}$$

where A is $\mathcal{R}$, ρ_α or $\mathcal{T}$ and B is R, w_β or T. The resulting generalized Konishi anomaly equations can be derived in the same way as above and can be summarized in the following matrix equation:

$$\begin{bmatrix} \hat{M}(z) & 2\langle T(z)\rangle & 0 \\ 0 & M(z) & 0 \\ 0 & 0 & M(z) \end{bmatrix}\begin{bmatrix} \langle T(z)T(w)\rangle_c & \langle T(z)R(w)\rangle_c & \langle T(z)w_\beta(w)\rangle_c \\ \langle R(z)T(w)\rangle_c & \langle R(z)R(w)\rangle_c & \langle R(z)w_\beta(w)\rangle_c \\ \langle w_\alpha(z)T(w)\rangle_c & \langle w_\alpha(z)R(w)\rangle_c & \langle w_\alpha(z)w_\beta(w)\rangle_c \end{bmatrix} =$$
$$= \frac{1}{3}G^2\partial_w\begin{bmatrix} T(z,w) & R(z,w) & 0 \\ R(z,w) & 0 & 0 \\ 0 & 0 & 5\epsilon_{\alpha\beta}R(z,w) \end{bmatrix} \tag{4.19}$$

Here we have used the chiral ring equations $G^2 w_\alpha = 0$ and $(G^2)^2 = 0$ as shown in the section 2. We have also dropped various connected 3-pt functions that vanish via eq. (3.13). $M(z)$ denotes the integral operator $(2\langle R(z)\rangle - I(z))$, $\hat{M}(z)$ denotes $M(z) - \frac{2}{3}G^2\langle T(z)\rangle$ and finally $R(z,w)$ and $T(z,w)$ denote $(\langle R(z)\rangle - \langle R(w)\rangle)/(z-w)$ and $(\langle T(z)\rangle - \langle T(w)\rangle)/(z-w)$ respectively.

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There are a few points to note about these equations:

1. Chiral ring equations are consistent with the above matrix equation. For example, if one takes the equation for $\langle w_\alpha(z)B(w)\rangle_c$ and multiplies by G^2 one finds that the equation identically vanishes in the chiral ring.

2. We have used the estimates given in the last section only to drop all the completely connected 3-pt. functions as they were shown to go as G^4. The estimates for the connected 2-pt. functions given in the Table 1, says that $\langle R(z)R(w)\rangle_c$, $\langle R(z)w_\alpha(w)\rangle_c$, and $\langle T(z)w_\alpha(w)\rangle_c$ all vanish in the chiral ring. We have not used these estimates in the above equation but we note that the (1,3), (2,2), (2,3), (3,1) and (3,2) matrix elements on the right hand side vanish. Thus the estimates given in the table are indeed consistent with the matrix equation above. In fact, in the next subsection, we will show that the solutions to this equation are unique thereby proving that these connected 2-pt. functions vanish. Similarly had we used the estimate for the 2-pt. function $\langle T(z)T(w)\rangle_c$ which was shown in the last section to go as G^2, we could have replaced $\hat{M}$ by M in the above equation.

3. The integrability condition is satisfied: the above equation is of the form

$$\mathcal{M}(z)N(z,w) = \partial_w K(z,w)\,, \tag{4.20}$$

where $\mathcal{M}(z)$ is the first matrix operator appearing on the left hand side of eq(4.19), $N(z,w)$ and $K(z,w)$ satisfy $N(z,w) = N^t(w,z)$ and $K(z,w) = K^t(w,z)$. The non-trivial consistency condition then is

$$(\partial_w K(z,w))\mathcal{M}^t(w) = \mathcal{M}(z)\partial_z K(z,w)\,. \tag{4.21}$$

The crucial identity needed for this is the one involving the integral operator $I(z)$ and is as follows:

$$(I(z)\partial_z - I(w)\partial_w)\frac{A(z)-A(w)}{z-w} = (\partial_z - \partial_w)\frac{I(z)A(z)-I(w)A(w)}{z-w} \tag{4.22}$$

for any function A which is smooth at z and w. This can be proved by using the definition of the contours involved in $I(z)$ and $I(w)$. It follows that the (1,2), (2,1) and (3,3) components of the integrability condition (4.21) implies the following equation:

$$G^2(\langle R(z)\rangle^2 - I(z)\langle R(z)\rangle) = 0\,. \tag{4.23}$$

This equation is just G^2 times the first equation of (4.6) if one takes into account the fact that the connected part of the correlation function appearing in the latter already is of order $(G^2)^2$. The only other non-trivial part of the integrability condition is its (1,1) component:

$$G^2[(2\langle R(z)\rangle - I(z))\langle T(z)\rangle - \frac{1}{3}G^2\langle T(z)\rangle^2] = 0 \tag{4.24}$$

which is just G^2 times the disconnected part of the third equation of (4.6) thereby

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proving the integrability condition for the (1,1) component. This is because all the connected parts appearing in that equation will be trivial when multiplied by G^2. Note also that the last term in eq. (4.24) could have been dropped as it is trivial. Its origin comes from the extra term in $\hat{M}$ appearing in the (1,1) component of $\mathcal{M}$ which as argued in the point 2 above could have been dropped.

4.2 Uniqueness of the solutions for the connected two point functions

Since the integrability conditions are satisfied, solution to eq. (4.19) exists. However the solution has a finite ambiguity which will be fixed by the physical requirement that the contour integrals around all the branch cuts of the connected two point functions in the the z and w planes must vanish separately. The reason for this is that the following operator equations hold:

$$\frac{1}{2\pi i}\int_{C_i} dzR(z) = S_i\,, \qquad \frac{1}{2\pi i}\int_{C_i} dwT(w) = N_i\,, \qquad \frac{1}{2\pi i}\int_{C_i} dzw_\alpha(z) = w_{\alpha\ i}\,, \quad (4.25)$$

where S_i is the chiral superfield whose lowest component is the gaugino bilinear in the i-th gauge group factor in the broken phase $\mathrm{U}(N) \to \prod_{i=1}^n \mathrm{U}(N_i)$ and $w_{\alpha\ i}$ is the U(1) chiral gauge superfield of the $\mathrm{U}(N_i)$ subgroup. Since these fields are background fields, in the connected correlation functions the contour integrals around the branch cuts must vanish. Similarly since these background fields are independent of the gravitational fields, order G^2 corrections to the one point functions of R, w_α and T must also have vanishing contour integrals around the branch cuts.

For later use, we can write a complete set of normalized differentials using eq. (4.7) upto order G^2 as

$$\omega_j = \frac{1}{4}\frac{dz}{[2\langle R(z)\rangle - V'(z)]}\frac{\partial}{\partial S_j}f(z)\,, \qquad \frac{1}{2\pi i}\int_{C_i}\omega_j = \delta_{ij}\,. \quad (4.26)$$

To illustrate the method, we will again focus on $\langle R(z)T(w)\rangle_c$. The action of operator $I(z)$ is given as:

$$I(z)\langle R(z)T(w)\rangle_c = V'(z)\langle R(z)T(w)\rangle_c + \sum_{k=0}^{n-1} c_k(w)z^k\,, \quad (4.27)$$

where the first term on the right hand side comes from the contour integral around z and the second term from that around ∞, with n being the order of $V'(z)$. Here we have used the fact that $R(z)$ asymptotically vanishes as $1/z$.[3] The coefficients $c_k(w)$ are arbitrary functions of w which asymptotically vanish as $1/w^2$. Similarly we have

$$I(w)\langle R(z)T(w)\rangle_c = V'(w)\langle R(z)T(w)\rangle_c + \sum_{k=0}^{n-1} \tilde{c}_k(z)w^k \quad (4.28)$$

with $\tilde{c}_k(z)$ being arbitrary functions of z that vanish asymptotically as $1/z^2$.

[3] Actually the coefficient of $1/z$ is $\mathrm{Tr}\, W^2$ and hence in the connected 2-pt function it vanishes. As a result the connected 2-pt function goes as $1/z^2$ (and for similar reasons $1/w^2$ asymptotically in w) which means that the sum over k is between 0 and $n-2$. However in the above expression we have kept the sum upto $n-1$ since, as it will turn out, the condition of vanishing contour integrals around all the branch cuts will in particular imply that $c_{n-1} = 0$

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From (4.19), the two equations that this two point function satisfies are as follows:

$$
\begin{aligned}
(2\langle R(z)\rangle - I(z))\langle R(z)T(w)\rangle_c &= \frac{1}{3}G^2\partial_w R(z,w) \\
(2\langle R(w)\rangle - I(w))\langle R(z)T(w)\rangle_c &= \frac{1}{3}G^2\partial_z R(z,w)
\end{aligned} \tag{4.29}
$$

The solutions to these two equations are

$$
\begin{aligned}
\langle R(z)T(w)\rangle_c &= \frac{1}{2\langle R(z)\rangle - V'(z)}\left[\frac{1}{3}G^2\partial_w R(z,w) + \sum_{k=0}^{n-1} c_k(w)z^k\right] \\
&= \frac{1}{2\langle R(w)\rangle - V'(w)}\left[\frac{1}{3}G^2\partial_z R(z,w) + \sum_{k=0}^{n-1} \tilde{c}_k(z)w^k\right].
\end{aligned} \tag{4.30}
$$

Equating the two right hand sides , we see that c_k and $\tilde{c}_k$ are not arbitrary functions of the respective arguments but are fixed upto a finite polynomial ambiguity in z as well as w. They must be of the form

$$
\begin{aligned}
\sum_{k=0}^{n-1} c_k(w)z^k &= \frac{1}{(2\langle R(w)\rangle - V'(w))}\frac{G^2}{3}\left[\langle R(w)\rangle\frac{(V'(w) - V'(z) + (z-w)V''(w))}{(z-w)^2} + \right. \\
&\qquad\qquad \left. + \frac{1}{4}\frac{(f(w) - f(z) + (z-w)f'(w))}{(z-w)^2} + \sum_{k,\ell=0}^{n-1} c_{k\ell}z^k w^\ell\right] \\
\tilde{c}_k(z) &= c_k(z) + \sum_\ell (c_{k\ell} - c_{\ell k})z^\ell\,,
\end{aligned} \tag{4.31}
$$

where $c_{k\ell}$ are arbitrary coefficients to be determined later. In deriving the above equation we have used the equation (4.7) with the right hand side set to zero. This is because the correction coming from the right hand side is of order G^4 and hence trivial.

Substituting this expression in eq. (4.30) and repeatedly using eq. (4.7), we obtain after some algebra:

$$
\begin{aligned}
\langle R(z)T(w)\rangle_c &= \frac{G^2}{3}\frac{1}{[2\langle R(z)\rangle - V'(z)][2\langle R(w)\rangle - V'(w)]} \times \\
&\times\left[\sum_{k,\ell=0}^{n-1} c_{k\ell}z^k w^\ell + \frac{[\langle R(z)\rangle\langle R(w)\rangle - V'(z)\langle R(w)\rangle - \frac{1}{4}f(z) + (z\leftrightarrow w)]}{(z-w)^2}\right].
\end{aligned} \tag{4.32}
$$

Note that the second term in the bracket is symmetric in $z \leftrightarrow w$.

As mentioned earlier, the connected two point function must obey the following conditions:

$$
\int_{C_i} dz\langle R(z)T(w)\rangle_c = \int_{C_i} dw\langle R(z)T(w)\rangle_c = 0 \tag{4.33}
$$

for all i, where C_i is the contour around the i-th branch cut.

Let us first consider contour integrals around the branch cuts in w-plane. To this end we can use the first equation in (4.30). The first term on the right hand side is a total derivative in w and therefore its contribution to the contour integral vanishes. Thus we arrive at the condition:

$$\int_{C_i} c_k(w) = 0\,. \tag{4.34}$$

Using the expression (4.31) for c_k, and the fact that $w^\ell/(2\langle R(w)\rangle + V'(w))$ for $\ell = 0,\dots,n-1$ form a complete basis of holomorphic 1-forms in the present case, these equations determine $c_{k\ell}$ completely. Very explicitly if

$$\begin{aligned}\sum_{k=0}^{n-1} t_k^{(i)} z^k &= \int_{C_i} dw \frac{1}{(2\langle R(w)\rangle - V'(w))} \times \\ &\times \left[\langle R(w)\rangle \frac{(V'(w) - V'(z) + (z-w)V''(w))}{(z-w)^2} + \frac{1}{4}\frac{(f(w) - f(z) + (z-w)f'(w))}{(z-w)^2}\right]\end{aligned} \tag{4.35}$$

then using the basis of normalized differentials eq. (4.26),

$$\sum_{k,\ell=0}^{n-1} c_{k\ell} z^k w^\ell = -\frac{1}{4}\sum_{i=1}^{n}\sum_{k=0}^{n-1} t_k^{(i)} z^k \frac{\partial}{\partial S_i} f(w)\,. \tag{4.36}$$

We will now show that $c_{k\ell}$ are symmetric in k and ℓ exchange. Let us define a matrix G_{ij} by the following equation

$$G_{ij} = \frac{1}{2\pi i}\int_{C_j} dz \frac{1}{2\langle R(z)\rangle - V'(z)} \sum_{k=0}^{n-1} t_k^{(i)} z^k\,. \tag{4.37}$$

We first simplify eq. (4.35) by using (4.7) as

$$\sum_{k=0}^{n-1} t_k^{(i)} z^k = -\int_{C_i} dw \frac{1}{(2\langle R(w)\rangle - V'(w))} \frac{2V'(z)V'(w) + f(z) + f(w)}{4(z-w)^2}\,, \tag{4.38}$$

where we have omitted a total derivative term with respect to w since it does not contribute to the contour integral. Substituting this in eq. (4.37) and noting that the residue of the first order pole $1/(z-w)$ vanishes due to eq. (4.7), we find that G_{ij} is symmetric. eq. (4.37) can be solved explicitly for $t_k^{(i)}$ as

$$\sum_{k=0}^{n-1} t_k^{(i)} z^k = \frac{1}{4}\sum_{j=1}^{n} G_{ij} \frac{\partial}{\partial S_j} f(z)\,. \tag{4.39}$$

Plugging this equation in eq. (4.36) and using the fact that G_{ij} is symmetric, we find that the coefficients $c_{k\ell}$ are symmetric in k and ℓ exchange as claimed above. .

Finally note that the symmetry of $c_{k\ell}$ implies that $\langle R(z)T(w)\rangle_c$ is symmetric in z and w.This will be crucial in the following. In particular this also implies that the contour integrals around the branch cuts in the z-plane vanish.

To summarize this subsection, although we have discussed in detail the example of $\langle R(z)T(w)\rangle_c$, it is easily seen that the eq. (4.19) and the conditions like eq. (4.33) fix the solutions for all the connected two point functions uniquely.

4.3 Solutions for the connected two point functions

Note that the right hand side of equation (4.19) vanishes for all components except $(1,1)$, $(1,2)$, $(2,1)$ and $(3,3)$ (the last being proportional to $\epsilon_{\alpha\beta}$). The uniqueness of the solution then implies

$$\langle R(z)R(w)\rangle_c = \langle w_\alpha(z)T(w)\rangle_c = \langle w_\alpha(z)R(w)\rangle_c = \langle w_{(\alpha}(z)w_{\beta)}(w)\rangle_c = 0\,. \tag{4.40}$$

This is in accordance with the estimates given in the Table 1. We have already obtained the solution for $\langle R(z)T(w)\rangle_c$ in equations (4.32), (4.35), (4.36). Let us denote this solution as $G^2H(z,w)$.

The remaining two equations are:

$$\begin{aligned} M(z)\langle w^\alpha(z)w_\alpha(w)\rangle_c &= \frac{10}{3}G^2\partial_w R(z,w) \\ M(z)\langle T(z)T(w)\rangle_c + 2\langle T(z)\rangle\langle R(z)T(w)\rangle_c &= \frac{1}{3}G^2\partial_w T(z,w)\,. \end{aligned} \tag{4.41}$$

Comparing the first equation with that of $\langle R(z)T(w)\rangle_c$ namely eq. (4.29) we conclude that

$$\langle w^\alpha(z)w_\alpha(w)\rangle_c = 10G^2H(z,w)\,. \tag{4.42}$$

Finally taking the derivative of eq. (4.29) with respect to $N_i\frac{\partial}{\partial S_i}$, and using the fact that to the leading order (i.e. order(1)) $\langle T(z)\rangle = (N_i\frac{\partial}{\partial S_i} + \frac{1}{2}w_i^\alpha w_{\alpha\ j}\frac{\partial^2}{\partial S_i\partial S_j})\langle R(z)\rangle$, we obtain the following equation:

$$M(z)N_i\frac{\partial}{\partial S_i}\langle R(z)T(w)\rangle_c + 2\langle T(z)\rangle\langle R(z)T(w)\rangle_c = \frac{1}{3}G^2\partial_w T(z,w)\,, \tag{4.43}$$

where we have used the fact that $G^2w_{\alpha\ i}$ is trivial. This equation is the same as the second equation of (4.41). Uniqueness of the solution then implies that upto order G^2,

$$\langle T(z)T(w)\rangle_c = G^2N_i\frac{\partial}{\partial S_i}H(z,w)\,. \tag{4.44}$$

We are now in a position to compute the gravitational corrections to the one point functions of $R(z)$, $w_\alpha(z)$ and $T(z)$ from eq. (4.6). Note that in this equation the two point functions contain both the disconnected and connected pieces. Since $\langle R(z)R(z)\rangle_c$ and $\langle R(z)w_\alpha(z)\rangle_c$ vanish, the first two equations do not contain any connected parts. Uniqueness then implies that one point function of R and $w_\alpha = w_\alpha^i\frac{\partial}{\partial S_i}R$ do not get any gravitational correction. The non-trivial equation is the third one. Using the results of this subsection we get

$$(M(z) - \frac{1}{3}G^2\langle T(z)\rangle)\langle T(z)\rangle + 12G^2H(z,z) = 0\,. \tag{4.45}$$

Expanding $\langle T(z)\rangle = T^{(0)} + G^2T^{(1)}$, with $T^{(0)} = N_i\frac{\partial}{\partial S_i}R + \frac{1}{2}w_i^\alpha w_{\alpha j}\frac{\partial}{\partial S_i}\frac{\partial}{\partial S_j}R$ we obtain

$$M(z)T^{(1)}(z) + \left[\frac{1}{3}(T^{(0)}(z))^2\right] + 12H(z,z) = 0\,. \tag{4.46}$$

Here the term indicated in the square bracket goes as N^2 (note that only the N_i dependent term in $T^{(0)}$ contribute since $G^2 w_\alpha$ is trivial) and therefore represents genus 0 contribution. On the other hand the term proportional to $H(z,z)$ does not come with any factors of N_i as is seen from the explicit solution given in (4.32), (4.35) and (4.36). This contribution therefore comes from genus 1. Writing $T^{(1)} = T_0^{(1)} + T_1^{(1)}$ where the subscript denotes the genus, we have the following solution to the above equation:

$$\begin{aligned} T_0^{(1)}(z) &= -\frac{1}{6} N_i \frac{\partial}{\partial S_i} T^{(0)}(z) \\ T_1^{(1)}(z) &= -\frac{12}{(2R^{(0)}(z) - V'(z))} [H(z,z) + c^{(1)}(z)] \end{aligned} \tag{4.47}$$

where $c^{(1)}(z)$ is a polynomial of degree $n-2$ and is uniquely determined by the requirement that the contour integrals of $T_1^{(1)}(z)$ around every branch cut vanishes. In the next subsection we will show that this is exactly the answer the Matrix model provides.

Let us note that the genus 0 contribution $T_0^{(1)}$ above can be absorbed in $T^{(0)}$ by a field redefinition

$$S_i \to S_i + \frac{1}{6} G^2 N_i \,. \tag{4.48}$$

In particular this means that the contribution of $T_0^{(1)}$ to the effective superpotential can be absorbed by the above field redefinition into the original genus 0 effective superpotential in the absence of the gravitational field. This also implies that this term does not contribute to the superpotential when evaluated at the classical solution of S_i in agreement with the statement made in [5].

4.4 Comparison with the matrix model results

In the Matrix model a systematic approach to computing higher genus contributions has been developed in [11, 12], however in the following we will rederive their results in a way parallel to the gauge theory discussion above. This will make the comparison between the two very transparent. Consider a hermitean matrix model with action given by $S = \frac{\hat{N}}{g_m} \sum_k \frac{g_k}{k} \mathrm{Tr} M^k \equiv \frac{\hat{N}}{g_m} V$, where M is a hermitean $\hat{N} \times \hat{N}$ matrix. In Matrix model the resolvent $\Omega(z) \equiv \frac{g_m}{\hat{N}} \mathrm{Tr} \frac{1}{z-M}$ satisfies a loop equation similar to gauge theory $R(z)$.

$$\langle \Omega(z) \rangle^2 - I(z) \langle \Omega(z) \rangle + \langle \Omega(z) \Omega(z) \rangle_c = 0 \,. \tag{4.49}$$

Here $I(z)$ is the same integral operator as in the gauge theory discussion above. In the large $\hat{N}$ limit, the two point function factorizes. However in the subleading order in $1/\hat{N}^2$ the connected part of the two point function (in fact the planar connected graph) contributes and which in turn yields the genus 1 contribution to the resolvent via the above equation. By definition

$$\begin{aligned} \langle \Omega(z) \Omega(w) \rangle_c &= \frac{1}{\hat{N}^2} \sum_{k=0}^{\infty} \frac{k}{z^{k+1}} \frac{\partial}{\partial g_k} \langle \Omega(w) \rangle \\ &\equiv \frac{1}{\hat{N}^2} \mathcal{O}(z) \langle \Omega(w) \rangle \,. \end{aligned} \tag{4.50}$$

Let us expand the 1-point function as :

$$\langle\Omega(z)\rangle = \Omega_{(0)}(z) + \frac{1}{\hat{N}^2}\Omega_{(1)}(z) + \cdots\,, \tag{4.51}$$

where dots represent terms of higher order in $1/\hat{N}^2$. Inserting these expansions in the above equations we get:

$$\begin{aligned} \Omega_{(0)}(z)^2 - I(z)\Omega_{(0)}(z) &= 0 \\ (2\Omega_{(0)}(z) - I(z))\Omega_{(1)}(z) + \mathcal{O}(z)\Omega_{(0)}(z) &= 0\,. \end{aligned} \tag{4.52}$$

Now we need to solve for $\mathcal{O}(w)\Omega_{(0)}(z)$. This can be done by applying the differential operator $\mathcal{O}(w)$ on the first equation of (4.52). To this end we need the following identity:

$$\mathcal{O}(w)V'(y) = \sum_{k=1}^{\infty}\frac{k}{w}(\frac{y}{w})^k = \frac{1}{(w-y)^2} \tag{4.53}$$

which is valid for $|w| > |y|$. It follows that

$$\begin{aligned} \int_{C_z} dy[\mathcal{O}(w)V'(y)]\frac{\Omega_{(0)}(y)}{y-z} &= \int_{C_z,|y|<|w|} dy\frac{1}{(w-y)^2}\frac{\Omega_{(0)}(y)}{y-z} \\ &= \partial_w\frac{\Omega_{(0)}(z)-\Omega_{(0)}(w)}{z-w}\,. \end{aligned} \tag{4.54}$$

Using this, we obtain the following equation by applying $\mathcal{O}(w)$ on the first equation of (4.52)

$$(2\Omega_{(0)}(z) - I(z))\mathcal{O}(w)\Omega_{(0)}(z) - \partial_w\frac{\Omega_{(0)}(z)-\Omega_{(0)}(w)}{z-w}\,. \tag{4.55}$$

Since $\Omega_{(0)}$ of the matrix model is the same as the $R^{(0)}$ for the gauge theory, we see that $\mathcal{O}(w)\Omega_{(0)}(z)$ satisfies the same equation (4.29) as $3\langle R(z)T(w)\rangle_c$. We now impose the conditions

$$\int_{C_i} dz\mathcal{O}(w)\Omega_{(0)}(z) = \int_{C_i} dw\mathcal{O}(w)\Omega_{(0)}(w) = 0 \tag{4.56}$$

which are the analogues of the equations (4.33). It follows from the discussion of uniqueness that $\mathcal{O}(w)\Omega_{(0)}(z)$ is equal to $3H(z,w)$. Note that as we showed in the last subsection, $H(z,w)$ is symmetric in z and w. This is consistent with the fact that $\mathcal{O}(w)\Omega_{(0)}(z)$ is symmetric in z and w. Finally, substitution of $\mathcal{O}(z)\Omega_{(0)}(z)$ in the second equation of (4.52), results in an equation for $\Omega_{(1)}(z)$ which is identical to that for the genus 1 part of the gauge theory equation (4.46) for $T_1^{(1)}$. Using the fact that the integral of $\Omega_{(1)}(z)$ around every branch cut is zero, we conclude, from the uniqueness of the solution, that

$$\Omega_{(1)}(z) = \frac{1}{4}T_1^{(1)}(z)\,, \tag{4.57}$$

with the right hand side being the genus 1 part of the solution given in (4.47). While in the matrix model the $\frac{1}{\hat{N}^2}$ correction to the effective potential is obtained by integrating the asymptotic expansion of $\Omega_{(1)}(z)$ with respect to the couplings g_k, the order G^2 correction to the effective superpotential in gauge theory is obtained by integrating the asymptotic expansion of $T_1^{(1)}(z)$ with respect to the coupling constants g_k (we already argued in the last subsection that the genus 0 contribution coming from $T_0^{(1)}$ can be absorbed by a field redefinition of S_i). eq. (4.57) implies therefore that the genus 1 contribution to the effective potential in matrix model is equal to the genus 1 contribution to the order G^2 term in the gauge theory effective superpotential. In fact, the relative coefficient 4 in eq. (4.57) is exactly reproduced if one follows the numerical factors in the diagrammatic computations given in [5].

5. Conclusions

In this paper we have analyzed the problem of computing the first non-trivial gravitational corrections to the effective superpotential, $\Gamma_1 = \int d^4x d^2\theta G^2 F_1(S)$, resulting from integrating out an adjoint scalar superfield with polynomial tree level superpotential.and minimal coupling to $N = 1$ supergravity. Whereas the problem has been earlier analyzed by [5] who showed, by using diagrammatic techniques, that the effective superpotential is in fact given by the genus one correction to free energy in the corresponding matrix model, we have here considered the issue from the point of view of "loop equations" arising from the generalized Konishi anomaly in presence of an $N = 1$ gauge W_α and supergravity $G_{\alpha\beta\gamma}$ background superfields, whose lowest components are the gaugino and the gravitino field strength respectively. It should be possible to generalize these methods to the case of other classical groups, as well as different representations for the chiral fields [13, 14]

From this point of view, the appearance of non-planar (genus one) diagrams is related to the lack of the usual factorization property of gauge invariant correlators in the presence of a non-trivial supergravity background, The fact that a non-trivial supergravity background deforms the chiral ring was stressed and used explicitly in the work by [5], where the failure of the factorization property was implicit. In this paper we have exploited both these facts and included also the supergravity contribution to the Konishi anomaly. First, we have estimated various two-and three- point connected correlators using the modified chiral ring relations, then we have derived loop equations which, due to the absence of factorization, involve connected two-point functions of "resolvents" $R(z)$, $T(z)$ and $w_\alpha(z)$, generalizing the equations obtained by [7] in two ways: first because we found additional contributions of order G^2, and second because we found it useful to consider two-point functions involving operators at two different points in the complex plane. We have then shown that these loop equations satisfy quite non-trivial consistency conditions and that there is a unique solution for the non-vanishing connected two-point functions. Using these results, we have then solved for the $\mathcal{O}(G^2)$, genus one, correction to the superpotential and found agreement with the matrix model result. We also find at $\mathcal{O}(G^2)$, a genus zero contribution to the superpotential, which can be removed by a field redefinition.

Concerning the issue of factorization (i.e. position independence) of chiral correlators in a supergravity background, we have observed in section 3 that our results do not contradict the known fact that this property holds in $N = 1$ gauge theories on Kähler manifolds (with non-trivial twisting if the manifold is not Hyper-Kahler) [10], since we indeed find that for purely bosonic background connected correlators vanish.

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Finally, it is a non-trivial open problem to extend the above results to higher genera. Indeed, whereas the chiral ring deformation that we have here employed follows directly from standard $N = 1$ supergravity tensor calculus. it has been shown by [4], using diagrammatic techniques, that in order to capture these higher order corrections one has to modify the chiral ring relations rather drastically, i.e. one has to include a self-dual two form $F_{\alpha\beta}$, which is a remnant of the graviphoton field strength of the parent $N = 2$ supergravity. The modified relation, $\{W_\alpha, W_\beta\} = 2G_{\alpha\beta\gamma}W^\gamma + F_{\alpha\beta}$,. does not have a conventional interpretation in $N = 1$ supergravity. It would be interesting to see how this non-standard modification can be incorporated in the approach followed here, based on generalized Konishi anomaly relations.

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A. Conventions

In this paper we follow conventions of [8]. Raising and lowering of spinor indices are done by the ϵ tensor as follows

$$\begin{aligned} W^\alpha &= \epsilon^{\alpha\beta}W_\beta\,, & W_\alpha &= \epsilon_{\alpha\beta}W^\beta\,, \\ \epsilon^{\alpha\beta}\epsilon_{\beta\alpha} &= 2\,, & \epsilon^{\alpha\beta}\epsilon_{\beta\alpha'} &= \delta^\alpha_{\alpha'}\,. \end{aligned} \tag{A.1}$$

We also define the following products of the $N = 1$ gauge multiplet and the $N = 1$ Weyl multiplet

$$\begin{aligned} W^2 &= W^\alpha W_\alpha = -W_\alpha W^\alpha\,, \\ G^2 &= G^{\alpha\beta\gamma}G_{\alpha\beta\gamma}\,, \qquad G_{\delta\gamma\alpha}G^{\delta\gamma}{}_\beta = -G_{\delta\gamma\beta}G^{\delta\gamma}{}_\alpha = \frac{\epsilon_{\alpha\beta}}{2}G^2\,. \end{aligned} \tag{A.2}$$

B. The gravitational contribution to Konishi anomaly

In this appendix we fix the normalization constant of the pure gravitational contribution to the the Konishi anomaly which was used in (4.5). The Konishi anomaly equation including the pure gravitational contribution in superspace is given by [15, 16]

$$\bar{D}^2(\bar{\Phi}e^V\Phi) = \frac{1}{32\pi^2}\,\mathrm{Tr}_{\mathrm{Ad}}(W^2) + \alpha\frac{N^2}{32\pi^2}G^2\,. \tag{B.1}$$

Here α is the unknown normalization constant which we will fix below. The θ^2 component of the above equation together with its anti-holomophic counterpart should reduce to the familiar equation of the chiral anomaly including the gravitational contribution given below.

$$\partial_\mu(\bar{\psi}\bar{\sigma}^\mu\psi) - \partial_\mu(\psi\sigma^\mu\bar{\psi}) = \frac{1}{32\pi^2}\frac{i}{2}\epsilon^{mnlk}\mathrm{Tr}_{\mathrm{Ad}}(F_{mn}F_{lk}) + \frac{N^2}{32\pi^2}\frac{1}{24}\frac{i}{2}\epsilon^{mnlk}R_{mnst}R^{st}{}_{lk}\,. \tag{B.2}$$

The coefficients in the above equation have been obtained from [17] Our strategy to fix the normalization constant α will be to extract the contribution of $R \wedge R$ from the superspace equation in (B.1) and require it to agree with the coefficient in (B.2). From [8] the lowest component of the $N = 1$ Weyl multiplet starts off as the gravitino field strength and is given by

$$G_{\alpha\beta\gamma} = \frac{1}{12}(\sigma^{ab}_{\alpha\beta}\psi_{ab\gamma} + \sigma^{ab}_{\beta\gamma}\psi_{ab\alpha} + \sigma^{ab}_{\gamma\alpha}\psi_{ab\beta})\,, \tag{B.3}$$

where we have set the auxiliary field in the above formula to zero, as we are working on shell and a, b refer to the local Lorentz indices. The gravitino field strength is defined by

$$\psi^{\alpha}_{ab} = \hat{D}_a\psi^{\alpha}_b - \hat{D}_b\psi^{\alpha}_a\,, \qquad \hat{D}_a\psi^{\alpha}_b = \partial_a\psi^{\alpha}_b + \psi^{\beta}_b\omega^{\alpha}_{n\beta}\,, \tag{B.4}$$

where $\omega^{\alpha}_{n\beta}$ is the spin connection. As we are working with an on shell background, equations of motion for the gravitino imply $\sigma^{ab}_{\alpha\beta}\psi_{ab\gamma} = \sigma^{ab}_{\alpha\gamma}\psi_{ab\beta}$. Therefore on shell, the lowest component of the Weyl multiplet is given by

$$G_{\alpha\beta\gamma} = \frac{1}{4}\sigma^{ab}_{\alpha\beta}\psi_{ab\gamma}\,. \tag{B.5}$$

The supersymmetric transformation on the gravitino field strength is given by

$$\delta\psi^{\alpha}_{mn} = -\xi^{\beta}R_{mnab}\sigma^{ab\alpha}_{\beta} + \cdots\,. \tag{B.6}$$

The dots in the above equation all refer to terms that involve the fermions, which are not of interest for the present purpose of determining the coefficient of $R \wedge R$. Substituting this variation in (B.5) we obtain

$$\delta G_{\alpha\beta\gamma} = \frac{1}{4}\sigma^{ab}_{\alpha\beta}\sigma^{cd}_{\delta\gamma}\xi^{\delta}R_{abcd}\,. \tag{B.7}$$

The θ^2 component of G^2 contains the $R \wedge R$ term, which is imaginary and thus contributes to the anomaly. This is given by

$$G^2|_{\theta^2} = \frac{i}{2}\frac{1}{16}\epsilon^{a'b'ab}R_{a'b'cd}R^{cd}_{ab}\,. \tag{B.8}$$

The total contribution to the anomalous current is obtained by subtracting this out with the $G_{\dot\alpha\dot\beta\dot\gamma}$, the anti-holomorphic contributions. Therefore the coefficient of the $R \wedge R$ term from W^2 is $1/8$. Comparing with the coefficient of $R \wedge R$ in (B.2) we see that $\alpha = 1/3$ in (B.1) in order to reproduce the chiral anomaly, including the gravitational contribution.

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